Black Holes in a Swirling Universe

R. Capobianco, B. Hartmann, N. Vas, J. Kunz, and J. Novo, *Photon rings and shadows of Kerr black holes immersed in a swirling universe*, Phys. Rev. D 113 no.6, 064053 (2026)

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Overview

- Rotation is seen in all celestial objects of our universe
- Finding exact solution to the Einstein field equations is difficult
- The Ernst Formalism allows us to generate new exact stationary and axially symmetric solutions via a transformation rather than directly solving Einstein's equations
- Recently, a new solution was generated using this technique [Astorino et al., 2022]
- The resulting metric describes a Kerr black hole immersed in a swirling universe (KBHSU), the properties of which we have investigated recently [Capobianco et al., 2026]

Image Credit: NASA, ESA, and the Hubble Heritage Team (STScI/AURA)

Stationary and Axisymmetric Spacetimes

The Ernst Formalism

- The Ernst equation is equivalent to the Einstein field equations for stationary and axially symmetric spacetimes
- The most general form of a stationary and axially symmetric metric is the (electric) Lewis–Weyl–Papapetrou (LWP) metric in coordinates (t, ρ, z, ϕ), given by

$$ds^2 = -f(\,dt - w\,d\phi)^2 + f^{-1}\left[\rho^2\,d\phi^2 + e^{2\gamma}(\,d\rho^2 + \,dz^2)\right]$$

  where f, w and γ are all functions of ρ and z
- Alternatively, a double–Wick rotation $t \to i\varphi$ and $\phi \to i\tau$ yields an alternative version, dubbed the magnetic LWP

$$ds^2 = f(\,d\varphi - w\,d\tau)^2 + f^{-1}\left[-\rho^2\,d\tau^2 + e^{2\gamma}(\,d\rho^2 + \,dz^2)\right]$$

- By casting a seed metric in one of these forms, a new solution can be generated via a transformation

The Ernst Equation

The Ernst Formalism

- Solving the (vacuum) field equations $R_{\mu\nu} = 0$ is highly non-trivial
- For stationary and axisymmetric spacetimes, the vacuum field equations are equivalent to the much more manageable Ernst equation [Ernst, 1967]

$$(\mathrm{Re}\,\mathcal{E})\nabla^2\mathcal{E} = \nabla\mathcal{E}\cdot\nabla\mathcal{E}$$

  where $\mathcal{E} = f + ih$ is the Ernst potential and $h = h(\rho, z)$ is constructed from w
- Rather than directly solving the field equations, the Ernst potential can simply be transformed to generate new solutions

Ehlers Transformation

$$\mathcal{E}' = \frac{\mathcal{E}}{1 + ij\mathcal{E}} \quad \text{where} \quad j \in \mathbb{R}$$

Harrison Transformation

$$\mathcal{E}' = \frac{\mathcal{E}}{1 + \alpha\alpha^*\mathcal{E}} \quad \text{where} \quad \alpha \in \mathbb{C}$$

Generating New Solutions

The Ernst Formalism

	Harrison	Ehlers
Electric	Reissner–Nordström	Taub–NUT
Magnetic	Schwarzschild in a Melvin–magnetic universe	Schwarzschild in a swirling universe

Table: Ehlers and Harrison transformations on a Schwarzschild black hole [Di Pinto, 2024]

Take the Kerr solution as the seed metric and cast it in the magnetic LWP form

$$ds^2 = f(d\phi - w\,dt)^2 + f^{-1}\left[-\rho^2\,dt^2 + e^{2\gamma}(d\rho^2 + dz^2)\right],$$

where

$$\rho = \sqrt{\Delta}\sin\theta \quad \text{and} \quad z = (r - M)\cos\theta,$$

$$\Delta = r^2 - 2mr + a^2, \quad \Sigma = r^2 + a^2\cos^2\theta, \quad f = 1 - \frac{2mr}{\Sigma}, \quad w = \frac{2Mar\sin^2\theta}{\Sigma - 2Mr}, \quad \text{and} \quad e^{2\gamma} = \frac{\Sigma - 2Mr}{\Delta + (M^2 - a^2)\cos^2\theta}.$$

Applying the Ehlers transformation gives the following metric:

Kerr black hole in a swirling universe

$$ds^2 = \mathcal{F}^{-1}(d\phi - \omega dt)^2 + \mathcal{F}\left[-\rho^2 dt^2 + \Sigma\sin^2\theta\left(\frac{dr^2}{\Delta} + d\theta^2\right)\right]$$

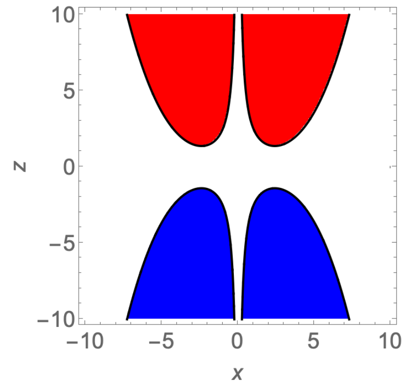
where $\mathcal{F}(r,\theta)$ and $\omega(r,\theta)$ are expressed in terms of the swirling parameter j

$$\mathcal{F} = \frac{\mathcal{F}_0 + j\mathcal{F}_1 + j^2\mathcal{F}_2}{R^2\Sigma\sin^2\theta}, \quad \omega = \frac{\omega_0 + j\omega_1 + j^2\omega_2}{\Sigma}$$

Ergoregions

Properties of the KBHSU

- Rotation induces frame dragging
- The swirling background induces two disconnected patches of ergoregions rotating oppositely above and below the equatorial plane (red retrograde, blue prograde)
- It has an odd-$\mathbb{Z}_2$ symmetry, as exemplified by its symmetrical ergoregions
- The Schwarzschild case $a = 0$ keeps the odd-$\mathbb{Z}_2$ symmetry
- In the Kerr case, the black hole's rotation also twists the spacetime so has its own ergosphere, breaking the symmetry
- As j increases, the three disconnected ergoregions begin to merge, and are fully merged after reaching the critical value $ajM = 0.25$



$M = 0$

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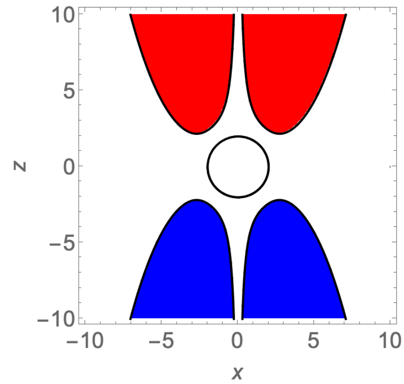

$a = 0, \; jM^2 = 0.1$

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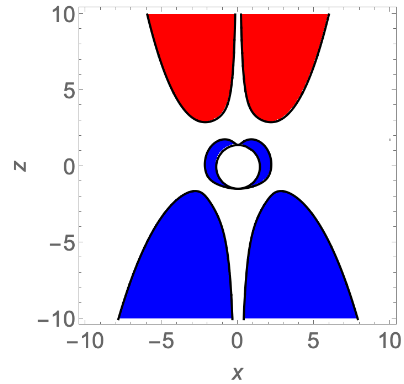

$a = 0.9M,\ jM^2 = 0.1$

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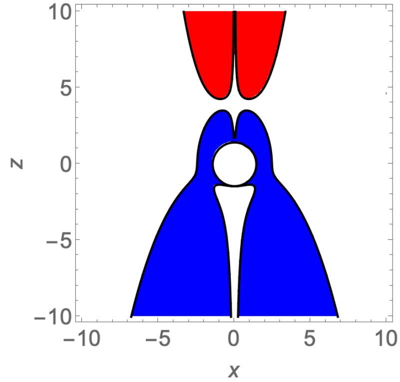

$a = 0.9M,\ jM^2 = 0.2$

Ergoregions

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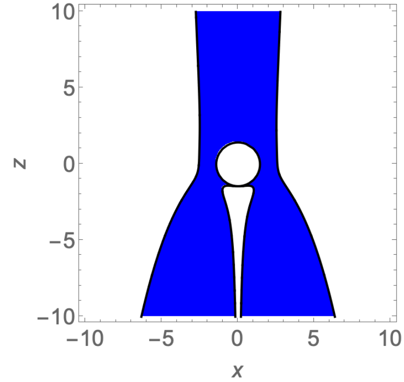


$a = 0.9M,\ ajM > 0.25$

Conical Singularities

Properties of the KBHSU

- For the axis to be smooth, the azimuthal angle must have period 2π, otherwise a conical singularity occurs
- Check this by considering the ratio between the perimeter and the radius of a small circle around the axis at the north and south poles:

$$\delta_0 = \frac{2\pi}{(1 - 4ajM)^2} \quad \text{and} \quad \delta_\pi = \frac{2\pi}{(1 + 4ajM)^2}$$

- Note the divergence at the critical value $ajM = \pm 0.25$
- Thus, at the critical value $ajM = 0.25$, there is a physical non-removable conical singularity along the upper half-axis

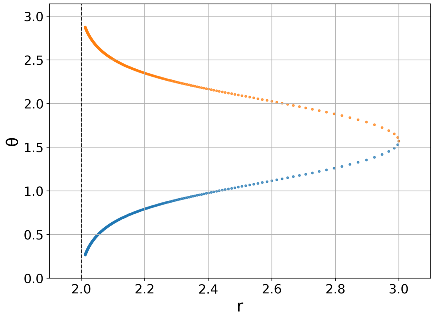
Light Rings

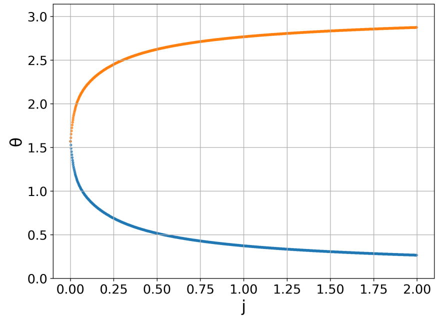
Properties of the KBHSU

- The coordinates (r, θ) of light rings are determined by the critical points of the potential $H_\pm$

$$H_\pm(r,\theta) = \frac{-g_{t\phi} \pm \sqrt{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}}{g_{\phi\phi}} = \omega \pm \mathcal{F}\rho$$

- For each parameter combination of a and j, two light rings exist, one associated with H_+ (•) and the other with H_- (•)





$a = 0$

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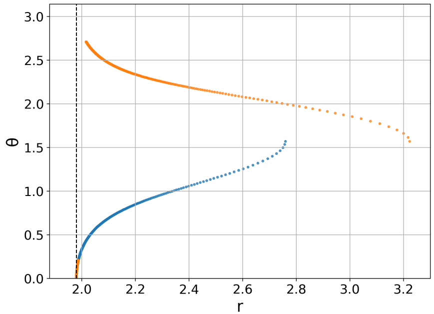
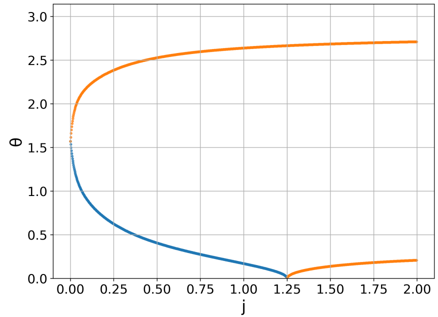
- For each parameter combination of a and j, two light rings exist, one associated with H_+ (●) and the other with H_- (●)





$a = 0.2M$

Light Rings

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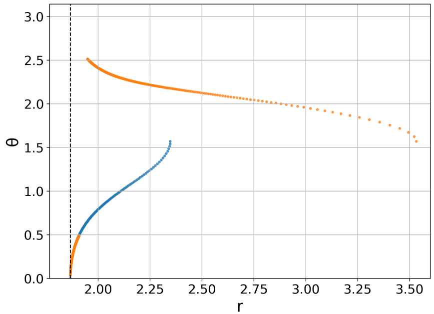
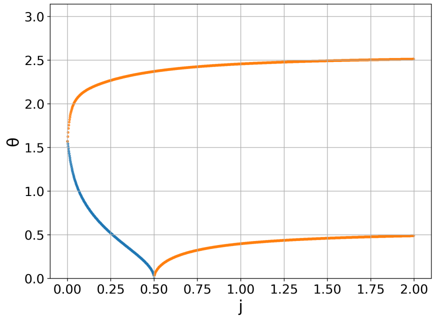
- For each parameter combination of a and j, two light rings exist, one associated with H_+ (●) and the other with H_- (●)





$a = 0.5M$

Light Rings

Properties of the KBHSU

- The coordinates (r, θ) of light rings are determined by the critical points of the potential $H_\pm$

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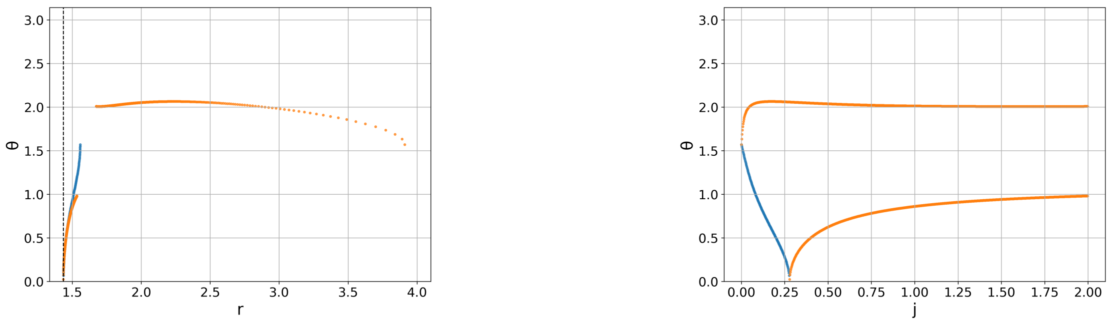
- For each parameter combination of a and j, two light rings exist, one associated with H_+ (●) and the other with H_- (●)



$a = 0.9M$

Light Points

Properties of the KBHSU

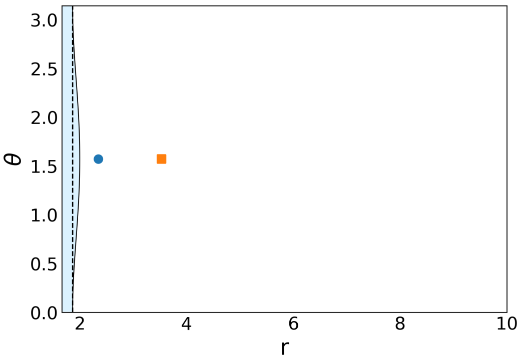
- The direction of light ring rotation is determined by the sign of the rotational velocity:

$$\Phi = \frac{d\phi}{dt}$$

- At each light ring, this quantity is given by

$$\Phi_\pm = \omega \pm \mathcal{F}\rho$$

- The outer light ring counter-rotates (■) to the black hole until it is captured by the ergoregion, where it starts to co-rotate (•) with the black hole
- At some point, the angular velocity must vanish
- This is a *light point* (∘) which occurs when the ergoregions merge



$a = 0.5M,\ j = 0$

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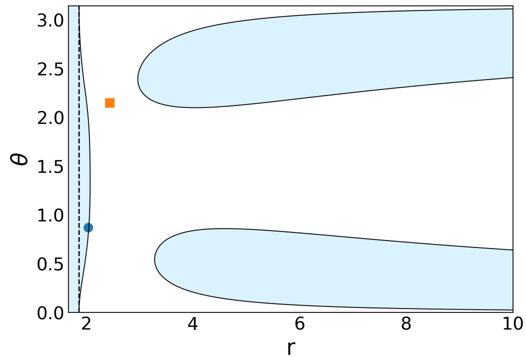

$a = 0.5M,\ jM^2 = 0.1$

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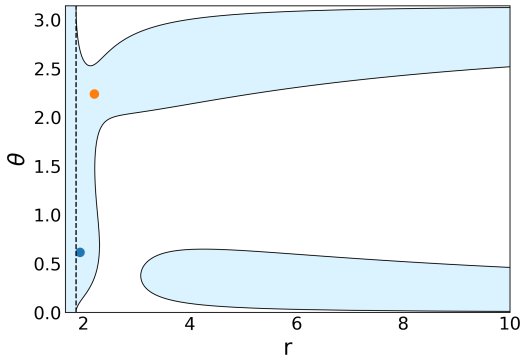

$a = 0.5M,\ jM^2 = 0.2$

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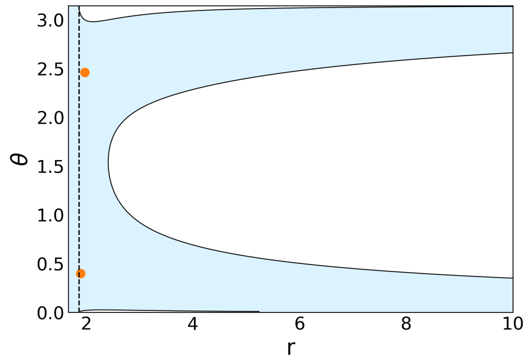

$a = 0.5M,\ jM^2 = 1$

Light Points

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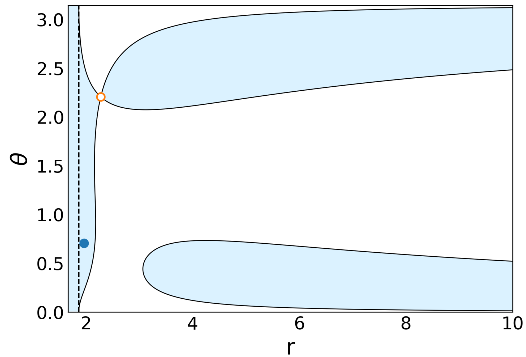
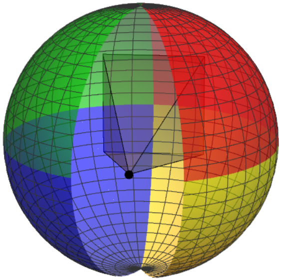
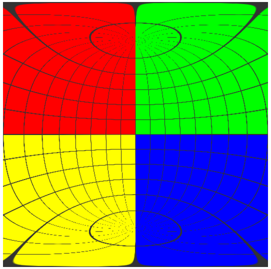


$a = 0.5M,\ jM^2 \approx 0.158579$

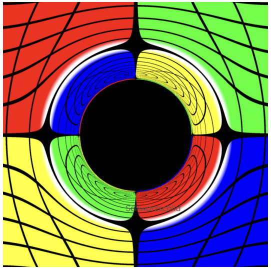
Shadows



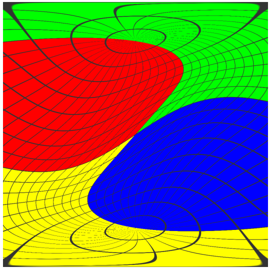
Schematic representation



Minkowski

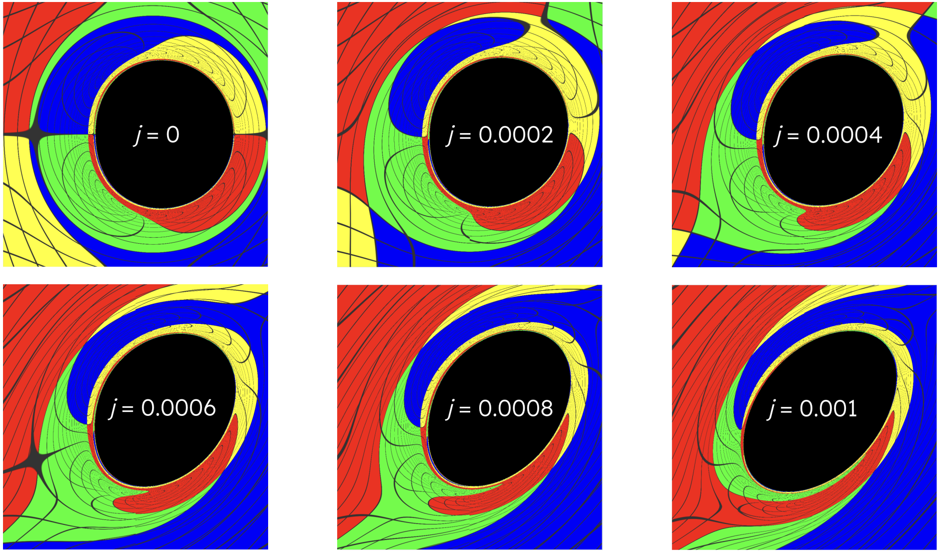


Schwarzschild



Swirling background

Shadows



$a = 0.9M$

Conclusions and Outlook

- It's very hard to find new solutions by directly solving the Einstein field equations, so transformations within the Ernst formalism can be employed to more easily generate new solutions
- The KBHSU is one such closed-form solution in general relativity
- The spin-spin interaction between the black hole's angular momentum and the swirling background leads to interesting effects such as the twisting of shadows
- In particular, the parameter combination of $ajM = 0.25$ is a critical value of the spacetime – presence of cosmic string?
- Other transformations have not been studied – Neugebauer–Bäcklund transformation of the Schwarzschild metric

References

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Thank you for listening!